

MOMENT MAPS AND NON-COMPACT COBORDISMS

Yael Karshon

Abstract

We define an *abstract moment map* for a torus action on a smooth manifold *without* a two-form. Cobordisms of such structures are meaningful even if the manifolds are noncompact, as long as the abstract moment maps are proper. We prove that a compact manifold with a torus action and an abstract moment map is cobordant to the normal bundle of its fixed point set. Two formulas follow easily: Guillemin's topological version of the abelian Jeffrey-Kirwan localization, and the Guillemin-Lerman-Sternberg formula for the Duistermaat-Heckman measure.

1. Introduction

This work is part of a joint project with Victor Guillemin and Viktor Ginzburg, in which we use cobordisms to study group actions on symplectic manifolds. Some of our results were announced and their proof sketched in [7].

In this paper we introduce a new, cleaner, cobordism technique. We treat non-compact manifolds directly, without “approximating” by compact manifolds as we did in [7]. Our basic theorem involves manifolds and not orbifolds; orbifolds only come up as reduced spaces. And we do not need to carry along a symplectic (or pre-symplectic) form for the theory to work.

Our basic object is a manifold with a torus action, a proper moment map, and an equivariant cohomology class. If (M, ω, Φ) is a symplectic manifold with a moment map in the usual sense, one can take the equivariant cohomology class represented by the sum $\omega + \Phi$. However, in our new approach we define an abstract moment map without a two-form (see section 2), and our equivariant cohomology class is unrelated to the

Received January 21, 1997, and, in revised form, December 26, 1997.