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CONVEX HULL PROPERTIES OF HARMONIC MAPS

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0. Introduction

In 1975, Yau [18] proved, by way of a gradient estimate, that a complete manifold M with non-negative Ricci curvature must satisfy the strong Liouville property for harmonic functions. The strong Liouville property (Liouville property) asserts that any positive (bounded) harmonic function defined on M must be identically constant. In 1980, Cheng [4] generalized the gradient estimate to harmonic maps from a manifold M with non-negative Ricci curvature to a Cartan-Hadamard manifold N. In particular, the Liouville property for harmonic maps can be derived for this situation. The Liouville property for harmonic maps asserts that if the image of the harmonic map is contained in a bounded set, then the map must be identically constant. In fact, Cheng's gradient estimate actually yields a slightly stronger theorem. It implies that if a harmonic map from a manifold with non-negative Ricci curvature into a Cartan-Hadamard manifold is of sublinear growth, then the map must be constant. A map $u : M \to N$ is of sublinear growth if there exist a point $p \in M$ and a point $o \in N$ such that the distance $d(u(x), o)$ between the image of u to the point o satisfies

$$d(u(x), o) = o(\rho(x)),$$

with $\rho(x)$ being the distance from $x \in M$ to p. Later, Kendall [10] proved that if a stochastically complete manifold satisfies the Liouville

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