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THE QUANTUM COHOMOLOGY OF BLOW-UPS OF $\mathbb{P}^2$ AND ENUMERATIVE GEOMETRY

L. GÖTTSCHE & R. PANDHARIPANDE

1. Introduction

The enumerative geometry of curves in algebraic varieties has taken a new direction with the appearance of Gromov-Witten invariants and quantum cohomology. Gromov-Witten invariants originate in symplectic geometry and were first defined in terms of pseudo-holomorphic curves. In algebraic geometry, these invariants are defined using moduli spaces of stable maps.

Let X be a nonsingular projective variety over $\mathbb{C}$. Let $\beta \in H_2(X, \mathbb{Z})$. In [13], the moduli space $\overline{M}_{0,n}(X, \beta)$ of stable n-pointed genus 0 maps is defined. This moduli space parametrizes the data $[\mu : C \to X, p_1, \ldots, p_n]$ where C is a connected, reduced, (at worst) nodal curve of genus 0, $p_1, \ldots, p_n$ are nonsingular points of C, and μ is a morphism. $\overline{M}_{0,n}(X, \beta)$ is equipped with n morphisms $\rho_1, \ldots, \rho_n$ to X where

$$\rho_i([\mu : C \to X, p_1, \ldots, p_n]) = \mu(p_i).$$

X is a convex variety if $H^1(\mathbb{P}^1, f^*(T_X)) = 0$ for all maps $f : \mathbb{P}^1 \to X$. In this case, $\overline{M}_{0,n}(X, \beta)$ is a projective scheme of pure expected dimension equal to

$$dim(X) + n - 3 + \int_\beta c_1(T_X)$$

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