

MEASURED SOLENOIDAL RIEMANN SURFACES AND HOLOMORPHIC DYNAMICS

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Abstract

We study, from a measure theoretic point of view, the lamination structure on the inverse limit space $\varprojlim(\bar{\mathbb{C}}, f)$ for an arbitrary rational map f on the sphere $\bar{\mathbb{C}}$. It turns out that there is an ergodic holomorphic foliated dynamical object $\mathcal{L}$, namely a self mapping of a measured solenoidal Riemann surface, which continuously injects into the inverse limit space, with full image and with leaves conformally isomorphic to the complex plane $\mathbb{C}$.

1. Introduction

The purpose of this note is to state and prove the following theorem which associates a holomorphic foliated dynamical object to an arbitrary rational map $f : \bar{\mathbb{C}} \rightarrow \bar{\mathbb{C}}$. The definitions are given below.

Theorem. *Given an arbitrary rational map $f : \bar{\mathbb{C}} \rightarrow \bar{\mathbb{C}}$ of degree $d \geq 2$ on the sphere $\bar{\mathbb{C}}$, there is an ergodic measured solenoidal Riemann surface $\mathcal{L}$ whose leaves are isomorphic to the complex plane $\mathbb{C}$, a holomorphic bijection $F : \mathcal{L} \rightarrow \mathcal{L}$, and a holomorphic map $\pi : \mathcal{L} \rightarrow \bar{\mathbb{C}}$ so that $f^n \circ \pi = \pi \circ F^n$ for every positive integer n . Moreover, the induced map of $\mathcal{L}$ into the reduced inverse limit space $\varprojlim(\bar{\mathbb{C}}, f)$ is a continuous injection whose image intersects every fiber in full measure for the natural multiplicity fiber measures class (see the proof in Section 4).*

Remarks and Definitions.

1. A topological solenoid, simply a solenoid, $\mathcal{S}$ is a topological space with local box charts homeomorphic to a product of the form (a

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