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MANIFOLDS ALL OF WHOSE FLATS ARE CLOSED

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Abstract

Let M be a locally irreducible Riemannian manifold such that (almost) any geodesic be contained in a compact immersed flat of dimension at least 2. The main result in this article implies that M is a locally symmetric space of the compact type. Therefore, our result gives a characterization of locally symmetric spaces of the compact type and higher rank in terms of flats.

0. Introduction

The purpose of this article is to give a higher rank rigidity characterization of compact locally symmetric spaces. Let us first begin with some motivations. Let M be a Riemannian manifold of nonpositive curvature with finite volume and $rank(M) = k$ in terms of Jacobi fields. Then any regular geodesic is contained in a k-flat, and hence the horizontal distribution, on the open set of regular vectors of TM, associated with the rank is integrable with totally geodesic leaves. (If M is not of nonpositive curvature, there seems to be no reason for expecting the integrability of the rank distribution and the rank is defined in terms of flats; cf. [12]). If M has in addition higher rank, i.e., $rank(M) \geq 2$, and irreducible universal cover, then M must be locally symmetric, as it follows from the celebrated theorem of Ballmann and Burns-Spatzier [1], [5] (see also [6], [8]). If M is not any more of nonpositive curvature, then the higher rank rigidity is not true. Namely, in [12] there is constructed a wide family of analytic (locally) irreducible compact nonsymmetric Riemannian manifolds of nonnegative curvatures and higher rank k, such that regular geodesics are contained in k-flats. (Some of these examples are homogeneous and were already known by Ernst

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