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RICCI FLOW AND THE UNIFORMIZATION ON COMPLETE NONCOMPACT KÄHLER MANIFOLDS

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1. Introduction

In the theory of complex geometry, the complete Kähler manifolds with positive holomorphic bisectional curvature have been studied for many years. If M is a complete compact Kähler manifold of complex dimension n with positive holomorphic bisectional curvature, people conjectured that M is biholomorphic to $\mathbb{CP}^n$. This was the famous Frankel Conjecture and was solved by Mori [34] and Siu–Yau [46] in 1979. In the case where M is noncompact, Greene–Wu [18] and Yau have the following conjecture:

Conjecture. *Suppose M is a complete noncompact Kähler manifold with positive holomorphic bisectional curvature. Then M is biholomorphic to $\mathbb{C}^n$.*

Several results concerning this conjecture were obtained in the past few years. In 1981, N. Mok, Y.T. Siu and S.T. Yau [31] proved the following theorem:

Theorem. *Suppose M is a complete noncompact Kähler manifold of complex dimension $n \geq 2$ with bounded and nonnegative holomorphic bisectional curvature. Suppose M is a Stein manifold. Suppose there exist constants $0 < \varepsilon, C_0, C_1 < +\infty$ such that*

(i) $\mathrm{Vol}(B(x_0,\gamma)) \geq C_0\gamma^{2n}, \; 0 \leq \gamma < +\infty,$

(ii) $0 \leq R(x) \leq \dfrac{C_1}{\gamma(x,x_0)^{2+\varepsilon}}, \; x \in M,$

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