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GAUSSIAN UPPER BOUNDS FOR THE HEAT KERNEL ON ARBITRARY MANIFOLDS

ALEXANDER GRIGOR'YAN

1. Introduction

In this paper, we develop a universal way of obtaining Gaussian upper bounds of the heat kernel on Riemannian manifolds. By the word "Gaussian" we mean those estimates which contain a Gaussian exponential factor similar to one which enters the explicit formula for the heat kernel of the conventional Laplace operator in $\mathbb{R}^n$:

$$p(x, y, t) = \frac{1}{(4\pi t)^{n/2}} \exp\left(-\frac{|x-y|^2}{4t} \right) .$$

The history of the heat kernel Gaussian estimates started with the works of Nash [25] and Aronson [2] where the double-sided Gaussian estimates were obtained for the heat kernel of a uniformly parabolic equation in $\mathbb{R}^n$ in a divergence form (see also [15] for the improvement of the original Nash's argument and [26] for a consistent account of the Aronson's results and related topics). In particular, the Aronson's upper bound for the case of time-independent coefficients which is of interest for us reads as follows:

$$p(x, y, t) \leq \frac{\mathrm{const}}{t^{n/2}} \exp\left(-\frac{|x-y|^2}{Ct} \right) ,$$

where C is a large enough constant.

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