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THE ATIYAH-JONES CONJECTURE FOR CLASSICAL GROUPS AND BOTT PERIODICITY

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0. Introduction

In this note we construct a *L-stratification* of $\mathcal{M}_k(G)$, the based G-moduli space of instantons (or equivalently anti instantons) of charge k over S^4. The structure groups we concern ourself with here are $SO(n)$ for $n > 6$ and $Sp(n)$. Then the general machinery of [6] is applied to prove

Theorem A. *For $G = SO(n)$ with $n > 6$ or $G = Sp(n)$ and for all $k > 0$ and all primes p, the Taubes inclusion map $\iota_k : \mathcal{M}_k(G) \longrightarrow \mathcal{M}_{k+1}(G)$ induces an isomorphism in homology*

$$(\iota_k)_t : H_t(\mathcal{M}_k(G); \mathbb{A}) \cong H_t(\mathcal{M}_{k+1}(G); \mathbb{A})$$

for $t \leq q = q(k) = [k/2] - 1$ and $\mathbb{A} = \mathbb{Z}$ or $\mathbb{Z}/p$.

The stratifications also lead naturally to the computation of the fundamental groups. We shall show that $\mathcal{M}_k(SO(n))$ are all simply connected for $n > 6$, and that the fundamental groups of $\mathcal{M}_k(SP(n))$ are always $\mathbb{Z}/2$. The argument of [6] can be applied to improve Theorem A as to state that ι_k induces a homotopy equivalence through dimension at least $[k/2] - 1$ for $SO(n), n > 6$ and at least $[k/2] - 2$ for $Sp(n)$. As a consequence of it, we are able to confirm the Atiyah-Jones conjecture that relates the homotopy of $\mathcal{M}_k$ to that of $\Omega_0^3 G$, a connected component of 3-fold loop space of group G (see [4]). To be precise, we shall prove

Theorem B. *For all positive integers k, the induced map (from ϑ_k)*

$$(\vartheta_k)_t : \pi_t(\mathcal{M}_k(G)) \longrightarrow \pi_t(\Omega_0^3 G)$$

on homotopy groups is an isomorphism for $t \leq q(k) = [k/2] - 1$ if $G = SO(n)$, $n > 6$; and for $t \leq q(k) = [k/2] - 2$ if $G = Sp(n)$.

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