

## INTEGRAL GEOMETRY OF PLANE CURVES AND KNOT INVARIANTS

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### Abstract

We study the integral expression of a knot invariant obtained as the second coefficient in the perturbative expansion of Witten's Chern-Simons path integral associated with a knot. One of the integrals involved turns out to be a generalization of the classical Crofton integral on convex plane curves, and it is related with the invariants of generic plane curves recently defined by Arnold, with deep motivations in symplectic and contact geometry. Quadratic bounds on these plane curve invariants are derived using their relationship with the knot invariant.

### 1. Introduction

The first and second order terms in the perturbative expansion of Witten's Chern-Simons path integral associated with a knot in the 3-space were first analyzed by Guadagnini, Martellini and Mintchev [8] as well as by Bar-Natan [4] shortly after Witten's seminal work. In an announcement which appeared in 1992, Kontsevich perceived a construction of a vast family of knot invariants which, presumably, contains the same information as the family of coefficients in the perturbative expansion of the Chern-Simons path integral associated with a knot [9]. In a recent paper [7], Bott and Taubes explored this construction in a much more detailed manner. At this stage, it seems that a rigorous foundation has been laid for studying the perturbative expansion of the Chern-Simons path integral associated with a knot. But, as it seems to us, we still lack a study of each individual knot invariant in this family in a way as concrete and thorough as possible. The first term in the perturbative expansion turns out to be a classical quantity associated with a space curve with nowhere vanishing curvature, which was studied extensively under the name of the Călugăreanu-Pohl-White self-linking

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