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SYMMETRIES OF FIBERED KNOTS

MONICA NICOLAU & SUSAN SZCZEPANSKI

Let $\mathscr{Z} \subset \mathbf{C}^{n+1}$ be an algebraic (analytic) hypersurface with an isolated singularity at the origin, which is given as the zero set of $f : \mathbf{C}^{n+1} \to \mathbf{C}$. Recall that the link of such a singularity (S^{2n+1}, K^{2n-1}) consists of a highly connected manifold K, embedded in the sphere S, as a codimension-two submanifold. Moreover, the complement $S - K$ of this embedding fibers over the circle, with the projection map given by the Milnor fibration $f(\mathbf{z})/|f(\mathbf{z})$. Thus these knots belong to a larger class of knots known as simple fibered knots. From one point of view, simple fibered knots are more general than the objects of study in spherical knot theory, since the submanifold K need not be a sphere; yet they are also more refined, since they are fibered knots.

Here we begin our investigation of finite cyclic actions on simple fibered knots (S^{2n+1}, K^{2n-1}) of dimension $n \geq 3$. Recall that a high dimensional knot is *simple* if its complement has the homotopy type of S^1 up to but not including its middle dimension. In particular, we consider simple fibered knots for which the submanifold K is a rational homology sphere. The more general situation, which requires modification of the proofs and techniques given here, as well as the introduction of some further invariants will be discussed in a separate paper [15]. We consider both the free and the semifree cases. We obtain a classification of both types of actions, as well as a determination of the number theoretic conditions which guarantee their existence.

We say that (S, K) admits a free $\mathbf{Z}_m$ action if $\mathbf{Z}_m$ acts freely on S leaving K invariant; we say that (S, K) admits a semifree action if the action on S is semifree with fixed set precisely K. Our results mirror those concerning spherical knots, found in [8], [13], [14], and [17], reflecting the fact that the objects of study are a generalization of these; the methods of proof necessarily address the nonvanishing of the homology of K and exploit the existence of the fibration of the complement.

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