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CONNECTION PRESERVING ACTIONS OF CONNECTED AND DISCRETE LIE GROUPS

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ABSTRACT. This paper examines connection preserving actions of a noncompact semisimple Lie group G on a compact fiber bundle and connection preserving actions of a lattice $\Gamma \subset G$ on a compact manifold. The results rely on a new technique that increases the regularity of sections of bundles naturally associated to the actions under consideration.

1. Introduction

Let M be a connected smooth n-dimensional manifold, and H a subgroup of $GL(n, \mathbb{R})$. An H-structure on M is a reduction of the full frame bundle over M to H. If we allow H to be a subgroup of $GL(n, \mathbb{R})^{(k)}$, the subgroup of k-jets at 0 of diffeomorphisms of $\mathbb{R}^n$ fixing 0, we can extend the notion of an H-structure to include reductions of higher order frame bundles to H. Given an H-structure $P \to M$, the automorphism group of P, $\mathrm{Aut}(P)$, is the subgroup of $\mathrm{Diff}(M)$ consisting of the diffeomorphisms of M whose induced action on the frame bundle preserves P. We wish to examine relationships between a Lie group G and manifolds M with H-structures such that $G \subset \mathrm{Aut}(P)$. Also, we are interested in the situation where, instead of a G action, we have only $\Gamma \subset \mathrm{Aut}(P)$, $\Gamma \subset G$ being a lattice subgroup. This case deals with the issue of the rigidity of the action of a higher rank lattice, an area of much recent research. The use of hyperbolic dynamical systems by Hurder in [7], and Katok and Lewis in [9] and [10] has produced recent results.

If we assume M is a compact manifold and G preserves a volume form on M, then the study of the ergodic theory of the action has been a successful technique in answering some of these questions. In particular, we mention Zimmer's work in [15] and [16] as examples of this technique. One drawback of this approach, however, is that the use of ergodicity provides measurable information which is often difficult to translate into meaningful information of a higher regularity. This information, which

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