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HYPERBOLIC MANIFOLDS WITH NEGATIVELY CURVED EXOTIC TRIANGULATIONS IN DIMENSION SIX

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0. Introduction

In this article we construct, given $\varepsilon > 0$, closed real hyperbolic manifolds of dimension 6 with exotic (smoothable) triangulations admitting Riemannian metrics with sectional curvatures in the interval $(-1, -\varepsilon, -1+\varepsilon)$.

A fundamental problem in geometry and topology is the following.

0.1. When are two homotopically equivalent manifolds diffeomorphic, PL homeomorphic, or homeomorphic?

When both manifolds in (0.1) are closed, hyperbolic, and of dimension greater than 2, Mostow's rigidity theorem states that they are isometric, in particular diffeomorphic. When both manifolds have strictly negative curvature, results of Eells and Sampson [4], Hartman [7], and Al'ber [1] show that if $f\colon M_1 \to M_2$ is a homotopy equivalence, then it is homotopic to a unique harmonic map. Lawson and Yau conjectured that this harmonic map is always a diffeomorphism (see problem 12 Yau [13], which asks for proof of (0.1), differentiably, for strictly negative curved manifolds). Farrell and Jones [5] gave counterexamples to this conjecture by proving the following. If M is a real hyperbolic manifold and Σ is an exotic sphere, then given $\varepsilon > 0$, M has a finite covering $\widetilde{M}$ such that the connected sum $\widetilde{M}\#\Sigma$ is not diffeomorphic to $\widetilde{M}$ and admits a Riemannian metric with all sectional curvatures in the interval $(-1-\varepsilon, -1+\varepsilon)$. Because there are exotic spheres only in dimensions 7 and up this does not give counterexamples to Lawson-Yau conjecture in dimension less than 7. The constructions here give counterexamples in dimension 6. Explicitly, we have the following theorem, that is a consequence of Theorem (3.1) and construction (3.2).

0.2. Theorem. *There are closed real hyperbolic manifolds M of dimension 6 such that the following holds. Given $\varepsilon > 0$, M has a finite cover*

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