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$SO(3)$-INVARIANTS FOR 4-MANIFOLDS WITH $b_2^+ = 1$. II

D. KOTSCHICK & J. W. MORGAN

1. Introduction

In this paper we extend the definition of Donaldson polynomial invariants to cover the case of manifolds with $b_1 = 0$ and $b_2^+ = 1$. We shall consider $SO(3)$-bundles with w_2 which lifts to an integral class. This paper generalizes [2] where the case of $SU(2)$-bundles with $c_2 = 1$ was treated. It should be viewed as the continuation of [5], where $SO(3)$-bundles with arbitrary p_1 were considered. It extends [5] in two ways. First of all, it completes the proof of the fact that the values of these invariants depend only on the chamber containing the self-dual harmonic 2-form for the metric used to define the anti-self-dual (ASD) equation. Secondly, it establishes more of the general properties of the differences of the values of the invariants as the self-dual 2-form crosses a wall. It follows from the properties that we establish here that, as conjectured in [5], the value of an invariant on every chamber is determined by its value on any one chamber; and in particular, the invariant is defined for all chambers regardless of whether they contain forms which are self-dual harmonic for some metric.

In spite of our progress, there is still more to be done, for we do not give an explicit formula in general, like the one in [2], for the difference term as the self-dual 2-form crosses a wall. We conjecture that there are systematic formulae for these difference terms involving only the classes defining the wall and the self-intersection form of the manifold.

Of course, as has been understood for a long time, the case of $b_2^+ = 1$ is unlike that of $b_2^+ > 1$ in that Donaldson invariants depend on the metric which is used to define the ASD equations. Naively, this gives invariants of Riemannian 4-manifolds. For applications one requires an understanding of the way the invariant depends on the metric. In the case $w_2 = 0$ and $p_1 = -4$ it was shown in [2] that the invariant only depends on the period point in the positive cone in $H^2(M; \mathbf{R})$ of the self-dual

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