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HOROSPHERIC FOLIATIONS AND RELATIVE PINCHING

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Abstract

Relative curvature pinching in negative curvature provides regularity of the horospheric foliations up to $C^{2-\epsilon}$.

The horospheric foliations of a negatively curved Riemannian manifold are defined as the stable and unstable foliations of its geodesic flow, as explained below. There are two classical results about smoothness of horospheric foliations: Negatively curved surfaces have C^1 horospheric foliations [4], and $\frac{1}{4}$-pinched Riemannian manifolds have C^1 horospheric foliations [2]. The latter has been improved to give $C^{2\sqrt{a}}$ foliations assuming a-pinching ($a \in (0, 1)$). An open question, posed in [2], is whether these results hold assuming only relative pinching (e.g., does relative $\frac{1}{4}$-pinching imply C^1 foliations). We do not know the answer, but give sufficient relative pinching conditions for the same range of smoothness and indicate where improvements seem possible. See [1] for a brief survey of interesting related results.

Definition 1. The sectional curvature of a compact negatively curved Riemannian manifold N is *relatively a-pinched* if $C \leq$ sectional curvature $< aC$ for some $C: N \to -\mathbb{R}_+$. If C is constant, the curvature is said to be (absolutely) a-pinched.

Theorem 2. *For* $a \in (0, 1)$ *a compact relatively a-pinched Riemannian manifold has* C^{2a} *horospheric foliations.*

This follows from Theorems 5 and 6. Theorem 5 is a regularity theorem for the stable and unstable foliations of an Anosov flow based on a "bunching" assumption of contraction and expansion rates sharpening the standard regularity theory in [1], which cannot be substantially improved. Theorem 6 establishes a connection between relative pinching and bunching which may not be optimal. Here are the needed properties

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