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ANALYTIC AND TOPOLOGICAL TORSION FOR MANIFOLDS WITH BOUNDARY AND SYMMETRY

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0. Introduction

Let G be a finite group acting on a Riemannian manifold M by isometries. We introduce *analytic torsion*

$$\rho_{\mathrm{an}}^G(M, M_1; V) \in \mathbf{R} \otimes_{\mathbf{Z}} \mathrm{Rep}_{\mathbf{R}}(G),$$

PL-torsion

$$\rho_{\mathrm{pl}}^G(M, M_1; V) \in K_1(\mathbf{R}G)^{\mathbf{Z}/2},$$

Poincaré torsion

$$\rho_{\mathrm{pd}}^G(M, M_1; V) \in K_1(\mathbf{R}G)^{\mathbf{Z}/2},$$

and *Euler characteristic*

$$\chi^G(M, M_1; V) \in \mathrm{Rep}_{\mathbf{R}}(G)$$

for ∂M the disjoint union of M_1 and M_2 and V an equivariant coefficient system. The analytic torsion is defined in terms of the spectrum of the Laplace operator, the PL-torsion is based on the cellular chain complex, and Poincaré torsion measures the failure of equivariant Poincaré duality in the PL-setting, which does hold in the analytic context. Denote by $\widehat{\mathrm{Rep}}_{\mathbf{R}}(G)$ the subgroup of $\mathrm{Rep}_{\mathbf{R}}(G)$ generated by the irreducible representations of real or complex type. We define an isomorphism

$$\Gamma_1 \oplus \Gamma_2 \colon K_1(\mathbf{R}G)^{\mathbf{Z}/2} \to (\mathbf{R} \otimes_{\mathbf{Z}} \mathrm{Rep}_{\mathbf{R}}(G)) \oplus (\mathbf{Z}/2 \otimes_{\mathbf{Z}} \widehat{\mathrm{Rep}}_{\mathbf{R}}(G))$$

and show under mild conditions that

$$\rho_{\mathrm{an}}^G(M, M_1; V) = \Gamma_1(\rho_{\mathrm{pl}}^G(M, M_1; V)) - \frac{1}{2} \cdot \Gamma_1(\rho_{\mathrm{pd}}^G(M, M_1; V)) + \frac{\ln(2)}{2} \cdot \chi^G(\partial M; V)$$

and

$$\Gamma_2(\rho_{\mathrm{pl}}^G(M, M_1; V)) = \Gamma_2(\rho_{\mathrm{pd}}^G(M, M_1; V)) = 0.$$

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