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DEFORMATIONS OF CONFORMAL STRUCTURES ON HYPERBOLIC MANIFOLDS

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Abstract

This paper deals with the geometry of group representations, namely with some geometric approach (from the viewpoint of the $(n+1)$-dimensional hyperbolic geometry) to the space of uniformized conformally flat structures on a hyperbolic n-manifold M of finite volume. In fact three kinds of deformations are studied: bendings, stampings, and stampings-with-torsion along totally geodesic submanifolds of M. The constructions of the last two deformations disprove a conjecture of C. Kourouniotis. The third kind of deformations yields at first time the existence of quasi-Fuchsian groups in space with "maximal" round conic domains in the discontinuous set. Also the problems of nonconnectivity and generation of the deformation space are discussed—they are related to results on the geometry of Nielsen hull and on nontrivial hyperbolic homology cobordisms in four dimensions.

1. Introduction

We will describe here some geometric approaches to the theory of deformations of conformal structures on a hyperbolic n-manifold M, $n \geq 3$, of finite volume, i.e., a complete Riemannian manifold M locally modelled on the hyperbolic (Lobachevsky) space $\mathbb{H}^n$ of constant sectional curvature -1.

The hyperbolic metric in $\mathbb{H}^n$ endows the $(n-1)$-dimensional sphere at infinite $S^{n-1} = \partial\mathbb{H}^n$ with a conformal structure, where the group $\operatorname{Isom}\mathbb{H}^n \cong O(n, 1)$ acts as the group of all conformal automorphisms. Taking the Poincaré ball model of the hyperbolic n-space (in the unit ball $B^n(0, 1) \subset \mathbb{R}^n$), we have the isomorphism (cf. [3]):

$$\{\mathbb{H}^n, \partial\mathbb{H}^n, O(n, 1)\} \cong \{B^n(0, 1), S^{n-1}, \text{Möb}(n-1)\},$$

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