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# LIMIT VOLUMES OF HYPERBOLIC THREE-ORBIFOLDS

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**Abstract**

We prove that the three smallest limit volumes for hyperbolic 3-orbifolds are $0.3053218\ldots$, $0.4444514\cdots$, and $0.4579827\ldots$. The corresponding unique orbifolds are given. We also show that an $n$-fold limit point of volumes of hyperbolic 3-orbifolds is bounded below by $(n - 1/2)v/2$ where $v = 1.01494\ldots$ is the volume of an ideal regular tetrahedron in hyperbolic 3-space. Applications to the order of the isometry groups of hyperbolic 3-manifolds are also given.

## 1. Introduction

A hyperbolic 3-orbifold is the quotient of hyperbolic 3-space by a discrete group of isometries of hyperbolic 3-space. If the group has no elliptic isometries, the quotient will be a hyperbolic 3-manifold. In all that follows, we will assume that the manifolds and orbifolds are all orientable.

The set of volumes of hyperbolic 3-manifolds are known to be well-ordered by the work of Thurston and Jørgensen (cf. [8]). In particular, given any specified set of hyperbolic 3-manifolds, there is a smallest volume among the set of volumes of the elements in the set. Results on the smallest volumes for hyperbolic 3-manifolds have been obtained in [1], [2], and [7].

Similarly, it is accepted folklore that the volumes of hyperbolic 3-orbifolds are well-ordered. A previous result on small volumes for hyperbolic 3-orbifolds was obtained by Meyerhoff in [6], where he found the smallest volume orientable cusped orbifold. However, unlike what occurs for orientable manifolds, the volume of this cusped orbifold is not a limit of volumes of closed orbifolds. Hence, it remains to find the smallest limit volume for a hyperbolic 3-orbifold.

In this paper, we show that a certain noncompact orbifold which is a quotient of the Borromean rings complement is the unique orbifold with the smallest volume that is a limit of volumes. Its volume is $0.3053218\ldots$. We also find the unique orbifolds with the second

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