J. DIFFERENTIAL GEOMETRY
33 (1991) 575–596

# THE RICCI FLOW ON 2-ORBIFOLDS WITH POSITIVE CURVATURE

LANG-FANG WU

**Abstract**

The Ricci flow on orbifolds converges asymptotically to a soliton solution. This also provides us with a canonical metric on every orbifold.

## 0. Introduction

Richard Hamilton [3] proved that under the Ricci flow a metric on any compact 2-dimensional, smooth manifold with positive curvature will converge to one of constant positive curvature. One can extend this result rather easily to 2-dimensional orbifolds whose universal covers are manifolds. In this paper we will prove an interesting result concerning the asymptotic behavior of the Ricci flow on the so-called class of "bad" orbifolds, or orbifolds whose universal cover is not a manifold.

**The Main Theorem.** *Any metric with positive curvature on a bad orbifold asymptotically approaches a Ricci soliton at time infinity under the Ricci flow, where a soliton is a solution which moves only by diffeomorphism.*

The main theorem gives us the first known example where a non-Kähler-Einstein orbifold converges to a nontrivial Ricci soliton, namely a metric of nonconstant curvature. The main theorem also provides us with a way to get a canonical metric on a bad orbifold. On a compact surface, there are no soliton solutions other than those of constant curvature (see Theorem 10.1 in [3]). Bad orbifolds do not admit metrics of constant curvature, so every soliton solution has nonconstant curvature. The main theorem also suggests strongly that a similar phenomenon may occur on higher dimensional Kähler manifolds.

A local coordinate expression of an equation on a manifold $M$ and on any quotient of $M$ by a finite group action look the same, since an orbifold is locally the quotient of a manifold by a finite group action. It is easy to obtain short time existence for the Ricci flow on an orbifold in

Received June 13, 1989 and, in revised form, November 27, 1989.