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AN EXPANSION OF CONVEX HYPERSURFACES

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Abstract

We study the motion of smooth, closed, uniformly convex hypersurfaces in a Euclidean $(n+1)$-space $\mathbf{R}^{n+1}$ expanding in the direction of their normal vectorfield with speed given by a suitable degree one homogeneous, positive, symmetric, concave function of the principal radii of curvature. We show that the hypersurfaces remain smooth and uniformly convex for all time and that asymptotically they become round.

1. Introduction

Let M_0 be a smooth, closed, uniformly convex hypersurface in a Euclidean $(n+1)$-space $\mathbf{R}^{n+1}$. Suppose that M_0 is given by a smooth embedding $X_0: S^n \to \mathbf{R}^{n+1}$. We consider the initial value problem

$$\begin{aligned}&\frac{\partial X}{\partial t}(x, t) = k(x, t)\nu(x, t),\\ &X(\cdot, 0) = X_0,\end{aligned} \tag{1.1}$$

where $k(\cdot, t)$ is a suitable curvature function of the hypersurface M_t parametrized by $X(\cdot, t): S^n \to \mathbf{R}^{n+1}$, and $\nu(\cdot, t)$ is the outer unit normal vectorfield to M_t.

Problems of this kind have been studied from several points of view. The motion of surfaces by their mean curvature was studied by Brakke [3] using the methods of geometric measure theory, while (1.1) with $k(\cdot, t) = -K(\cdot, t)$, where K is the Gauss curvature, was proposed by Firey [7] as a model for the wearing of stones on a beach by water waves.

More recently, Huisken [11] considered the case $k(\cdot, t) = -H(\cdot, t)$, where H is the mean curvature, and showed that in this case the initial value problem (1.1) has a unique smooth solution for some maximal time interval $[0, T)$, and as $t \to T$, the hypersurfaces M_t converge to a point P. Moreover, the hypersurfaces $\widetilde{M}_t$, obtained from M_t by a homothety

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