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A C^∞ SCHWARZ REFLECTION PRINCIPLE IN ONE AND SEVERAL COMPLEX VARIABLES

STEVE BELL & LÁSZLÓ LEMPERT

1. Introduction

The classical Schwarz Reflection Principle of one complex variable is a theorem about boundary behavior of holomorphic mappings. It is easy to state a reasonable analogue of the reflection principle in the C^∞ category, and not too hard to back the statement with a proof. In this paper, we will prove a version of a one variable C^∞ reflection principle in such a way that it generalizes naturally and with very little alteration to a setting in several complex variables.

The one variable C^∞ reflection principle that interests us here is the following.

Theorem 1. *Suppose that γ_1 and γ_2 are C^∞ smooth curves in the complex plane, and suppose there is a point $z_0 \in \gamma_1$ and a disc D centered at z_0 such that $D-\gamma_1$ consists of exactly two simply connected components, which we denote by D_+ and D_-. Suppose that there is a holomorphic function f defined on D_+, which extends continuously to γ_1 such that the extension maps γ_1 to γ_2. Then f extends C^∞ smoothly up to γ_1 near z_0. Furthermore, if f is not a constant function, there is a positive integer n such that $f^{(n)}(z_0) \neq 0$.*

In [5], Čirka proved the regularity part of this theorem and went on to prove more general results about mappings in several variables. In [11], Rosay extended Čirka's results to apply to a class of nonholomorphic mappings; Rosay's proofs, when viewed in the context of Čirka's original result, are simpler and more natural.

The finite order vanishing statement in the theorem was first proved by Alinhac, Baouendi, and Rothschild in [2].

The main result of this paper is an extension of this one variable theorem to a theorem about boundary regularity and uniqueness of holomorphic maps which map a hypersurface into a Levi flat hypersurface. The precise

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