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$L^{n/2}$-CURVATURE PINCHING

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The famous sphere theroem states that a complete, simply connected 1/4-pinched manifold is homeomorphic to the standard sphere [2], [22], [26]. It is also known that the homeomorphism theorem can be sharpened to a diffeomorphism theorem, if a more restrictive pinching condition is imposed [12], [27], [28], [20].

On the other hand, Gromov proved the negative pinching theorem provide the pinching constant also depending on the diameter of the manifold [14], [17].

In this paper we prove pinching theorems for $L^{n/2}$-curvature bounded Riemannian manifolds. We denote the norm of the curvature tensor $\mathrm{Rm}(g)$ of the metric g by $|\mathrm{Rm}(g)|$. Our main results may now be stated as follows.

Theorem A. *For any $i_0 > 0$, $H > 0$, and integer $n \geq 4$, there exists a constant $\mu = \mu(H, i_0, n) > 0$, such that if (M, g) is a complete Riemannian manifold with* $\operatorname{diam} M = n \geq 4$, *and*

(a) $\mathrm{Ric}(g) \geq -Hg$,

(b) $\mathrm{inj}(g) \geq i_0$,

(c)
$$\max_{x\in M} \int_{B_{i_0}(x)} |\mathrm{Rm}(g)|^{n/2}\, dg \leq H,$$

(d)
$$\max_{x\in M} \int_{B_{i_0}(x)} |R(g)_{ijkl} - (g_{ik}g_{jl} - g_{il}g_{jk})|^2\, dg \leq \mu,$$

then M is homotopic to a Riemannian manifold $\overline{M}$ of positive constant sectional curvature, in particular, M is compact. Furthermore, M is covered by a topological sphere.

Theorem B. *For each $H > 0$, $i_0 > 0$, $d > 0$, and integer $n \geq 4$, there exists a small constant $\mu = \mu(H, i_0, d, n) > 0$, such that if (M, g) is a*

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