J. DIFFERENTIAL GEOMETRY
32 (1990) 533-538

A REMARKABLE SYMPLECTIC STRUCTURE

LARRY BATES & GEORGE PESCHKE

Abstract

An explicit example of an exotic symplectic structure on R^4 is given.

1. Introduction

An old theorem due to Darboux [1] asserts that about each point x in any symplectic manifold (M, ω) there exists a neighborhood of x and a local chart (q^a, p_b) such that the symplectic form ω has the local representation

$$\omega = dq^a \wedge dp_a.$$

Naturally enough, one could try to make the domain of such a chart as large as possible. It can happen that one may not be able to enlarge the domain of such a symplectic chart beyond a certain size. This obstruction is of a geometric nature and has only come to light through recent work of Gromov [2].

To explain this, let ω_0 be the standard symplectic structure on R^{2n} and let $N \subset R^{2n}$ be any closed Lagrangian submanifold.

Theorem (*Gromov*). $[\omega_0] \neq 0$ *in* $H^2(R^{2n}, N; R)$, *the second relative de Rham cohomology group of the pair* (R^{2n}, N).

Being closed, ω_0 has a potential ψ on R^{2n}, i.e., $d\psi = \omega_0$. Furthermore, $[\psi|N] \neq 0$ in $H^1(N; R)$.

In this paper we explicitly endow a manifold M diffeomorphic to R^4 with a symplectic form ω admitting a Lagrangian torus T such that $[\omega] = 0$ in $H^2(M, T; R)$. But then Gromov's theorem tells us that (M, ω) does not symplectically embed in (R^4, ω_0). Thus ω is an exotic symplectic structure on M.

We note that the existence of exotic symplectic geometries was already known to Gromov [2], although the techniques used in the course of the proof do not permit explicit construction of an example.

Received June 2, 1989.