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COLLAPSING RIEMANNIAN MANIFOLDS WHILE KEEPING THEIR CURVATURE BOUNDED. II

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0. Introduction

This is the second of two papers concerned with the situation in which the injectivity radius at certain points of a riemannian manifold is "small" compared to the curvature.

In Part I [3], we introduced the concept of an F-*structure* of *positive rank*. This generalizes the notion of a torus action, for which all orbits have positive dimension. We showed that if a compact manifold, Y^n, admits an F-structure of positive rank, then it also admits a family of riemannian metrics, g_δ, whose sectional curvatures are uniformly bounded independent of δ and for which the injectivity radius, $i_y(g_\delta)$ goes uniformly to zero at all points $y \in Y^n$, as $\delta \to 0$. Such a sequence is said to *collapse with bounded curvature* (see Part I for variants and refinements of the above result).

In the present paper, we prove a kind of strengthened converse to the collapsing theorem. If $y \in Y^n$, let $|K(y)|$ denote the maximum of the absolute value of the sectional curvature over $\tau \in \Lambda^2(T_y(Y^n))$.

Theorem 0.1. *There exist constants $c_1(n), c_2(n) > 0$ such that if Y^n is a complete riemannian manifold, then $Y^n = Y^n_F \cup Y^n_G$, where*

(1) *Y^n_F is an open set which admits an F-structure of positive rank, whose orbits, $\mathscr{O}_y$, have diameter satisfying* $\operatorname{diam}(\mathscr{O}_y) \le c_1(n) i_y$,

(2) *for all $y \in Y^n_G$, there exists w in the ball $B_{i_y/c_2(n)}(y)$ with*

$$|K(w)|^{1/2} i_y \ge c_2(n). \tag{0.2}$$

Remark 0.3. For the F-structure we construct, the local actions almost preserve the metric. By applying Lemma 1.3 of [3], we can replace the metric on Y^n by a nearby metric which is invariant for the F-structure on Y^n_F.

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