J. DIFFERENTIAL GEOMETRY
31 (1990) 791-845

CONVEX REAL PROJECTIVE STRUCTURES ON COMPACT SURFACES

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Abstract

The space of inequivalent representations of a compact surface S with $\chi(S) < 0$ as a quotient of a convex domain in $\mathbb{RP}^2$ by a properly discontinuous group of projective transformations is a cell of dimension $-8\chi(S)$.

The purpose of this paper is to investigate convex real projective structures on compact surfaces. Let $\mathbb{RP}^2$ be the real projective plane and $\mathbf{PGL}(3, \mathbb{R})$ the group of projective transformations $\mathbb{RP}^2 \to \mathbb{RP}^2$. A *convex real projective manifold* (*convex $\mathbb{RP}^2$-manifold*) is a quotient $M = \Omega/\Gamma$, where $\Omega \subset \mathbb{RP}^2$ is a convex domain and $\Gamma \subset \mathbf{PGL}(3, \mathbb{R})$ is a discrete group of projective transformations acting properly on Ω. The universal covering of M may then be identified with Ω, and the fundamental group $\pi_1(M)$ with Γ. Two such quotients $M_1 = \Omega_1/\Gamma_1$ and $M_2 = \Omega_2/\Gamma_2$ are *projectively equivalent* if there is a projective transformation $h \in \mathbf{PGL}(3, \mathbb{R})$ such that $h(\Omega_1) = \Omega_2$ and $h\Gamma_1 h^{-1} = \Gamma_2$. The classification of convex $\mathbb{RP}^2$-manifolds with $\chi(M) \geq 0$ is due to Kuiper [30], [31] in early 1950's.

If S is a closed smooth surface, then a *convex $\mathbb{RP}^2$-structure* on S is defined to be a diffeomorphism $f: S \to M$ where M is a convex $\mathbb{RP}^2$-manifold; two such pairs (f, M) and (f', M') are regarded as equivalent if there is a projective equivalence $h: M \to M'$ such that $h \circ f$ is isotopic to f'. Let $\pi = \pi_1(S)$ by the fundamental group of S. Given a convex $\mathbb{RP}^2$-structure on S, the action of π by deck transformations on the universal covering space of S determines a homomorphism $\pi \to \mathbf{PGL}(3, \mathbb{R})$, well defined up to conjugacy in $\mathbf{PGL}(3, \mathbb{R})$. The set of

Received November 30, 1988. The author gratefully acknowledges partial support from National Science Foundation grant DMS-8613576 and an Alfred P. Sloan Foundation fellowship.