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# ON THE GENERALIZATION OF KUMMER SURFACES

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## 1. Introduction

The subject of this paper is on the compact complex $n$-manifolds with trivial canonical bundle. Complex tori are of course the obvious examples which have been extensively studied. In this paper, we shall go into the other types of manifolds which have the zero first Betti number. The typical examples for complex surfaces are $K3$ surfaces. On the 3-dimensional case, aside from the geometric point of view, part of the purpose for the study of this type of 3-folds comes from particle physics in the finding of the Ricci flat Kähler 3-folds with zero first Chern class and 1st Betti number. The topological invariants of such manifolds, especially their Euler numbers and the fundamental groups, are interesting for physicists [2]. In a previous paper [5], we had constructed a subclass of such 3-manifolds. In this paper, the analogous construction in the higher dimensional case is considered. The following is the general problem:

Find the projective manifolds with trivial canonical bundle and zero first Betti number by the construction of resolving singularities of the quotient of a complex torus by a finite group. Compute their Euler numbers.

We shall only consider the case when acting on the torus, the group is abelian and all the elements in this group have at least one common fixed point. Consider the complex torus $V$ to be a complex Lie group with 0 as the identity element. We may assume $G$ to be a finite abelian group of Lie-automorphisms of $V$, and also the dualizing sheaf $\omega_{V/G}$ to be trivial. If $x$ is an element of $V$ fixed by some nontrivial element of $G$, the action of its isotropic subgroup $G_x$ on $V$ near $x$ is isomorphic to that of some diagonal finite group $g$ acting on $\mathbb{C}^n$ near the origin $\vec{0}$. Inside the affine algebraic variety $\mathbb{C}^n/g$, $\mathbb{C}^{*n}/g$ ($\mathbb{C}^* \doteqdot \mathbb{C}\backslash\{0\}$) is a Zariski-open set which has the structure of an algebraic torus, denoted by $T$. Then $\mathbb{C}^n/g$ is an equivariant affine embedding of $T$. A toroidal desingularization of $\mathbb{C}^n/g$ means a nonsingular equivariant embedding $\widehat{\mathbb{C}^n/g}$ of $T$ together with a $T$-equivariant birational morphism $\pi\colon \widehat{\mathbb{C}^n/g} \to \mathbb{C}^n/g$. In this

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