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CR STRUCTURES OF CODIMENSION 2

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0. Introduction

There are several known methods of associating a Cartan connection on a principal bundle to a nondegenerate codimension 1 CR structure. By analyzing an appropriate moduli space, we are able to define a class of admissible codimension 2 CR structures which is analogous to the class of nondegenerate CR structures. Given an admissible CR structure on a manifold M we construct a principal bundle, in fact a subbundle of the frame bundle of M, and a connection on this bundle. In addition, we decompose TM as a direct sum of subbundles of fiber dimensions 1 and 2.

In §1 we present the basic definitions and some fundamental examples of CR structures. In §2, after defining the Levi map of a CR structure and the moduli space in which the Levi map is valued, we compare CR structures of codimensions 1 and 2 and then define admissible CR structures. In §3 and §4 we develop the geometry of such structures by methods reminiscent of both Chern's treatment of nondegenerate CR structures [1] and Webster's treatment of pseudohermitian structures [5]. In §5 we examine the moduli space in detail. Finally, in §6 we consider some examples of admissible CR structures, and raise a few unanswered questions. One need not master the intricacies of §5 in order to understand the discussion in §6.

Terminology is either standard or defined when it first appears. Any undefined differential geometric terms can be found in [2]. Several notational conventions are used, but not mentioned, in the text:

(1) There are a great number of indexed entities, and the range of the index varies with the entity. There are frequent reminders of the appropriate range, but there are also many instances when the range is given only implicitly, by the entity itself. The index set $\{1, 2, \cdots, n\}$ is denoted by I_n; the null set is sometimes denoted by I_0.

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