

SUTURED MANIFOLDS AND GENERALIZED THURSTON NORMS

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Over the past several years, David Gabai has developed new and powerful machinery for the study of 3-manifolds ([3]–[7]). Among the long-mysterious questions he has answered are the Poenaru conjecture, the Property R conjecture, the superadditivity of knot genus, and property P for satellite knots. It is the intention here to give an account of these developments, starting at the very beginning.

This began as an expository paper. It first became apparent that a generalization of the Thurston norm would be useful in such an exposition when it provided a direct alternative to an awkward doubling argument in [3]. Eventually it became clear that the theory of sutured manifolds could be developed from a different perspective, a perspective which is combinatorial, uses no foliations, is perhaps more general (cf. the latitude in the definition of index in 7.4), and seems to recover and extend (cf. [12]) the principal topological results. The absence of foliations simplifies the proof at several points and is perhaps interesting in its own right.

All of the central results of the paper were proven first by Gabai, with the exception of superadditivity of knot genus. This last theorem we proved simultaneously (announcements crossed in the mail). Gabai's proof uses the machinery of [4] (§5 here) and a trick of A. Thompson [13]. The proof here is an illustration of the use of β -norms for β a 1-complex more complicated than a circle (in fact an "eyeglass").

The paper is organized as follows: §§0 and 1 establish notation and describe the generalized Thurston norm. §2 describes sutured manifold decompositions and show that β -taut decompositions can be built on any relative homology class. §3 is essentially a careful proof of [3, Lemma 3.5]. Gabai's original theorem is only sketchily proved and the hypotheses incomplete. Since this theorem is central to the development here (it substitutes for foliations), we give a complete statement and detailed proof. §4 proves that sutured manifold