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ON THE AVERAGE INDICES OF CLOSED GEODESICS

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Introduction

A nonconstant closed curve $c\colon S^1 = \mathbf{R}/\mathbf{Z} \to M$ on a compact Riemannian manifold M with metric g is a closed geodesic on M iff it is a critical point of the energy functional $E\colon \Lambda M \to \mathbf{R}$, $E(c) = \frac{1}{2}\int_{S^1} g(\dot{c}, \dot{c})$ on the Hilbert manifold ΛM of closed curves (cf. [11, Chapter 1]). Due to a theorem of Lusternik and Fet there always exists a closed geodesic on a compact Riemannian manifold.

ΛM carries a canonical O(2)-action leaving E invariant. With a closed geodesic c all iterates c^m, $m \in \mathbf{N}$, with $c^m(t) = c(mt)$ are closed geodesics too. Two closed geodesics $c_1, c_2\colon S^1 \to M$ are geometrically distinct if their images $c_1(S^1)$ and $c_2(S^1)$ are distinct. D. Gromoll and W. Meyer prove in [6] that on a compact Riemannian manifold there are infinitely many geometrically distinct closed geodesics if the sequence $b_i(\Lambda M; F)$ of Betti numbers of ΛM w.r.t. a field F is unbounded. In [21] M. Vigue-Poirrier and D. Sullivan prove that for a compact simply-connected manifold the sequence $b_i(\Lambda M; \mathbf{Q})$ of rational Betti numbers of ΛM is bounded iff the cohomology algebra $H^*(M; \mathbf{Q})$ of M is a truncated polynomial algebra $T_{d,n+1}(x)$ with the generator x of degree d and height $n+1$.

If M is a compact rank-one symmetric space ("CROSS") then the sequence $b_i(\Lambda M; F)$ is bounded for any field F. In this case one can use the following result of W. Klingenberg and F. Takens (cf. [13], [11, 3.3]): For a C^4-generic metric on a compact manifold either there exists a nonhyperbolic closed geodesic of twist type (then a version of the Birkhoff-Lewis fixed point theorem due to J. Moser [18] implies the existence of infinitely many geometrically distinct closed geodesics) or all closed geodesics are hyperbolic. So far there is no example of a simply-connected compact Riemannian manifold with only hyperbolic closed geodesics. If M is a compact simply-connected manifold rational homotopy equivalent to a CROSS with a metric all of whose

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