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HANDLEBODIES AND p-CONVEXITY

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The aim of this paper is to study the Riemannian geometry of manifolds with boundary. In a previous paper [4], the author proved the following theorem.

Let M be a compact connected manifold with nonempty boundary. If M admits a Riemannian metric with nonnegative sectional curvature and p-convex boundary, then M has the homotopy type of a CW-complex of dimension $\leqslant p-1$.

Note. The author has recently learned that this theorem has also been proved independently by H. Wu [5].

One of the main results of this paper is a converse of this theorem.

We begin by recalling the notion of p-convexity. Let X be an $(n-1)$-dimensional (normally oriented) hypersurface in a Riemannian manifold Ω and let $\lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_{n-1}$ be its principal curvature functions. X is called p-convex if $\lambda_1 + \cdots + \lambda_p > 0$ at each point of X. Note in particular that "1-convexity" is the usual notion of convexity; "$(n-1)$-convexity" means that X has positive mean curvature. Also note that p-convexity implies $(p+1)$-convexity.

In [3], by a handle-attaching process, Lawson and Michelsohn showed the following: *Suppose X has positive mean curvature and let X' be a hypersurface obtained from X by attaching an ambient k-handle to the positive side of X. If the codimension $(n-k)$ of the handle is $\geqslant 2$, then X' can be constructed also to have positive mean curvature.* (That is to say that X' is ambiently isotopic to a hypersurface of positive mean curvature.)

Our central result is a generalization of this theorem to the p-convex case. Specifically we shall prove the following.

Theorem 1. *Let X be a (normally oriented) p-convex hypersurface in a Riemannian manifold Ω, and let X' be a hypersurface obtained from X by attaching a k-handle D^k to the positive side of X. If $k \leqslant p-1$, then X' can be constructed also to be p-convex.*

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