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INSTANTONS ON $\mathbf{CP}_2$

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0. Introduction

The purpose of this paper is to describe and classify the instantons on $\mathbf{CP}_2$ for the unitary and the classical compact simple Lie groups. The description closely resembles that given for the instantons on S^4 by Atiyah, Drinfeld, Hitchin and Manin [3], both being based on the bijective correspondence between instantons and holomorphic vector bundles on an associated complex manifold known as the Ward correspondence.

Although the problem of describing instantons can be converted into one in complex analysis and will be treated here strictly as such, its roots lie elsewhere and the background will now be briefly outlined.

Let X be an oriented 4-dimensional Riemannian manifold and G a compact Lie group. A *G-instanton* on X is a G-vector bundle F on X with G-connection ∇ such that the curvature F_∇ is *self-dual*: $* F_\nabla = F_\nabla$, where $*$ is the Hodge $*$-operator acting on 2-forms on X. In these circumstances, the *Yang-Mills equations* $\nabla * F_\nabla = 0$ are automatically satisfied by virtue of the Bianchi identity $\nabla F_\nabla = 0$, and solutions of the Yang-Mills equations are of considerable physical importance (see e.g. [1] or [17]).

The case $G = SU(2)$ is of particular interest from both a physical and a mathematical viewpoint. On the physical side, there is, for example, the well-known result that if X is spin, then the connection induced on the self-dual spin bundle by the Levi-Civita connection is self-dual iff X is Einstein. On the mathematical side, one has Donaldson's celebrated theorem [9] on the intersection forms of smooth compact 4-manifolds, the proof of which is based on topological properties of the space of $SU(2)$-instantons of second Chern class -1 on the 4-fold in question (endowed with a suitable metric). The existence of such instantons was proved by Taubes [19] amongst a number of other results; these will be returned to shortly.

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