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# GAUSS PARAMETRIZATIONS AND RIGIDITY ASPECTS OF SUBMANIFOLDS

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The normal spherical image, i.e., the Gauss map, plays a crucial role in the geometry of a euclidean hypersurface. In general, the Gauss map is not invertible. Our starting point here is the observation that whenever the rank (or the relative nullity) is constant, then one has a representation by the inverse of the Gauss map on the normal bundle of its image, which we call *Gauss parametrization*. This has many interesting applications. In particular, it is useful in the study of rigidity problems.

Recall the classical theorem of Beez-Killing: A hypersurface is locally rigid in $\mathbf{R}^{n+1}$ if the rank of the Gauss map is at least 3. A main result in this paper is that any complete minimal hypersurface $M^n$ in $\mathbf{R}^{n+1}$ is rigid as a minimal submanifold of $\mathbf{R}^{n+p}$ for any $p \geqslant 1$, provided $n \geqslant 4$, and $M$ is irreducible; cf. Theorem 2.1. This result is global in nature and fails to be true locally. In fact, any simply connected minimal hypersurface of rank 2 has precisely a one-parameter associated family of minimal deformations, already in codimension 1. Some other rigidity results will be discussed in the last section. Our main theorem there implies that (locally irreducible) hypersurfaces with nonzero constant mean curvature are locally rigid; cf. Theorem 3.3. In [4], we will deal with further applications of the Gauss parametrization, for example, a classification of real Kaehler hypersurfaces.

It seems that the idea of Gauss parametrizations has been used systematically only in a special case by Sbrana. In a beautiful paper [19], he studied deformable hypersurfaces about five years before E. Cartan considered the problem.

## 1. Gauss parametrizations

In this section, after reviewing some basic facts, we will discuss a local classification of hypersurfaces with constant relative nullity in spaces of constant curvature.

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