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CHAINS IN CR GEOMETRY

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Abstract

There is a well-defined system of curves on any nondegenerate CR manifold of hypersurface type. It is shown that any two sufficiently close points on a strictly pseudo-convex abstract CR hypersurface can be connected by a smooth curve from this family. Such a general result does not hold for other signatures.

This paper shows that any two sufficiently close points on a strictly pseudo-convex CR hypersurface can be connected by a smooth chain. For purposes of exposition we first present the complete proof for a three-dimensional submanifold of $\mathbf{C}^2$. Then in §4 we indicate the changes necessary to cover the general case of an abstract CR structure of hypersurface type. We work with C^∞ structures but it will be obvious that one only needs C^k, k large. The results are new even for real analytic structures and it is not clear if a shorter proof would be possible in this case.

A CR structure on a three-dimensional manifold M is a 2-plane distribution $H \subset TM$ together with a fibre preserving map J: $H \to H$ with $J^2 = -I$. Given such a structure, we may choose a real 1-form ω which annihilates H and a complex 1-form ω_1 which annihilates all vectors of the form $X + iJX$, $X \in H$. These choices can be done in such a way that $\omega \wedge \omega_1 \wedge \bar{\omega}_1$ is different from zero in a neighborhood of a given point. We are interested in results of a local nature on M so we may shrink M and assume $\omega \wedge \omega_1 \wedge \bar{\omega}_1$ is everywhere different from zero. Conversely, given ω and ω_1 with $\omega \wedge \omega \wedge \bar{\omega}_1 \neq 0$ we may easily construct H and J.

Any three-dimensional submanifold M of $\mathbf{C}^2$ has an induced CR structure. Let $\tilde{J}$: $T\mathbf{C}^2 \to T\mathbf{C}^2$ give the complex structure. Then $H = TM \cap \tilde{J}TM$ and $J = \tilde{J}|_H$. Note that if Φ: $U \to V$ is a biholomorphism of open sets in $\mathbf{C}^2$, then $M \cap U$ and $\Phi(M) \cap V$ have the same CR structure. The forms ω and ω_1 can

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