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ASYMPTOTIC BEHAVIOR OF CONVEX SETS IN THE HYPERBOLIC PLANE

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1. Introduction

In this note we study the following problem posed by L. A. Santaló and I. Yañez in [5]:

"*Let $C(t)$ be a family of bounded closed convex sets in the hyperbolic plane, depending on the parameter t $(0 \leqslant t)$ and such that $C(t_1) \subset C(t_2)$ for $t_1 < t_2$. Assume that for any point P of the plane there is a value t_P of t such that, for all $t \geqslant t_P$, we have $P \in C(t)$. We then say that $C(t)$ expands over the whole plane as $t \to \infty$. If $F(t)$ and $L(t)$ denote respectively the area and length of $C(t)$, prove that*:

$$\lim_{t \to \infty} \frac{L(t)}{F(t)} = (-K)^{1/2},$$

where $K < 0$ is the curvature of the hyperbolic plane".

The quotient area length appears in a natural way in problems of classical geometric probability. For instance, given a compact convex set in the Euclidean plane we can consider the length of the intersection of a random straight line (in the sense of the integral geometry) with this convex set. In this way we obtain a random variable σ whose expected value $E(\sigma)$ is

$$E(\sigma) = \pi F/L.$$

If we expand now this convex over the whole Euclidean plane we have (cf. [6])

$$\lim_{t \to \infty} \frac{L(t)}{F(t)} = 0$$

and so the expected value of σ tends to infinity.

L. A. Santaló and I. Yañez remark in [5] that the situation in the hyperbolic plane is quite different. In fact they prove, using the hyperbolic isoperimetric inequality and the Gauss-Bonnet formula, that for a family of sets convex

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