J. DIFFERENTIAL GEOMETRY
19 (1984) 517-560

SELF-DUAL CONNECTIONS ON 4-MANIFOLDS WITH INDEFINITE INTERSECTION MATRIX

CLIFFORD HENRY TAUBES

Abstract

Let M be a compact, connected and oriented Riemannian 4-manifold. Sufficient conditions on M and a principal SU(2) bundle $P \to M$ are established which imply that P admits a smooth, irreducible, self-dual connection.

1. The main results

This article addresses the following question: When does a principal SU(2) bundle, P, over a smooth, compact, oriented, 4-dimensional Riemannian manifold, M, admit an irreducible, self-dual connection? In a previous article [20], the author established that if the intersection matrix (cf. [15])

$$Q\colon H_2(M;\mathbf{Z}) \times H_2(M;\mathbf{Z}) \to \mathbf{Z}$$

of M is positive definite, then a necessary and sufficient condition on P is that the second Chern class of $P \times_{\mathrm{SU}(2)} C^2$ satisfy $c_2(P) < 0$. This article extends the existence results to manifolds with indefinite intersection matrix. The main result here is that a principal SU(2) bundle $P \to M$ admits a smooth, irreducible self-dual connection whenever $-c_2(P)$ is large with respect to $b_- = \frac{1}{2}(\operatorname{rank}(Q) - \operatorname{signature}(Q))$. These results are stated in detail in Theorems 1.1 and 1.2 below.

A number of the results which are stated below were deduced independently by S. K. Donaldson, to whom the author is greatly indebted for many invaluable discussions.

This article should be considered as a sequel to [20], where most of the notation and terminology is introduced. The reader may find the expositions in

Received March 15, 1983 and, in revised form, January 30, 1984. This work was completed while the author was a guest at the Mathmatical Institute, 24-29 St. Gilles, Oxford OX1-3LB, England, and a Junior Fellow of the Harvard University Society of Fellows and was supported in part by the National Science Foundation under grant #PHY-82-03669.