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PATH-CONNECTED YANG-MILLS MODULI SPACES

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Abstract

Min-max techniques in the calculus of variations are used to prove that the moduli spaces of self-dual connections on principal SU(2) or SU(3) bundles over S^4 are path-connected.

1. Introduction

On a principal bundle $P \to S^4$ whose structure group, G, is a compact, simple and simply connected Lie group, there are distinguished connections. These are the connections whose curvature is self-dual with respect to the Hodge dual of the metric on T^*S^4 which is induced from the identification $S^4 = \{x \in \mathbf{R}^5 \colon |x|^2 = 1\}$. (This metric is called the standard metric.)

The moduli space of self-dual connections on P,

$$\mathfrak{M}(P) = (P_s \times \{\text{smooth, self-dual connections on } P\})/\text{Aut } P,$$

is a smooth manifold. Here P_s is the fibre of P at s = south pole, and Aut P is the group of smooth automorphisms of P. The isomorphism class of P is specified by its integer degree, $k(P)$ [4]. (For $G = \mathrm{SU}(2)$, $k(P) = -c_2(P \times_{\mathrm{SU}(2)} \mathbf{C}^2)$.) If $k(P) < 0$, then $\mathfrak{M}(P) = \varnothing$; if $k(P) = 0$, then $\mathfrak{M}(P) =$ point; and if $k(P) > 0$, then $\mathfrak{M}(P)$ is nontrivial.

Although these spaces have been the subject of much recent study, [4], [11], [14], relatively little is known of their global structure. A small advance is made in this article with the following theorem.

Theorem 1.1. *Let $P \to S^4$ be a principal $G =$ SU(2) or SU(3) bundle with positive degree. Then $\mathfrak{M}(P)$ is path-connected.*

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