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A FIRST EIGENVALUE ESTIMATE FOR MINIMAL HYPERSURFACES

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0. Introduction

In this paper we obtain a lower bound for the first eigenvalue of a compact orientable hypersurface embedded minimally in a compact orientable manifold with positive Ricci curvature. The proof of our result utilizes Reilly's formula [4] which is actually an integrated version of Bochner's formula, as it becomes clear in the proof of Theorem 1. It should be mentioned that the idea of integrating Bochner's formula was used before by Lichnerowicz [3].

Combining our theorem with the result of Yang and Yau [5] we obtain an upper bound for the area of an embedded minimal surface of S^3 solely in terms of its genus. See Proposition 4 for a more general statement. We also estimate an upper bound of the length of closed geodesics in a convex surface.

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1. Reilly's formula

Let Ω be a Riemannian manifold of dimension n with smooth boundary $\partial\Omega$. Let f be a function defined on Ω which is smooth up to $\partial\Omega$. We denote $z = f|_{\partial\Omega}$ and $u = (\partial f/\partial n)|_{\partial\Omega}$, where $\partial f/\partial n$ is the outward normal derivative of f. $\overline{\Delta} f$ and $\overline{\nabla} f$ denote the Laplacian and the gradient of f with respect to the Riemannian metric of Ω, whereas Δg and ∇g denote the Laplacian and the gradient of g (defined on $\partial\Omega$) with respect to the induced Riemannian metric on $\partial\Omega$. Let $X, Y \in T_p\Omega$. Then we define the Hessian tensor $(\overline{D}^2 f)(X, Y) = X(Yf) - (\overline{D}_X Y)f$ where X and Y are extended arbitrarily to a vector field near

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