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# RIEMANNIAN MANIFOLDS WITH BOUNDED CURVATURE RATIOS

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## 1. Introduction

The following classical result of Schur is well-known: If the sectional curvature of a riemannian manifold of dimension greater than two at every point is the same for every element of the grassmannian of tangent 2-planes, then the curvature is constant over the manifold. It is natural to ask what can be said if the ratios of sectional curvatures are close to one only. Gribkov [3] shows that not much can be said for open manifolds. He proves that even if the ratios of the curvatures are arbitrarily close to one, the variation of the curvature over the manifold can still be arbitrarily large.

In this paper we prove that if the riemannian manifold is compact, the sectional curvature is positive, and the curvature ratios are close to one, then the manifold is diffeomorphic to a spherical space form. This result is new even if we specialize to simply connected manifolds and assert the existence of a homeomorphism only. The well-known sphere theorem of Berger [1] and Klingenberg [4], while optimal in other respects, does not apply under the above local pinching assumption.

Basically, the result is a consequence of the second Bianchi equation and the Calderón-Zygmund inequality. We prove that the curvature $R$ is close to a certain local average $\bar{R}$ whose covariant derivative is small. Techniques similar to those of [5] provide a new metric connection $\nabla'$ on $M$ whose curvature $R'$ is close to $\bar{R}$ and whose torsion $T'$ is small. As a consequence, [5, Theorem 2] applies and yields the result.

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