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ON THE EXISTENCE OF CODIMENSION-ONE MINIMAL SPHERES IN COMPACT SYMMETRIC SPACES OF RANK 2. II

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Introduction

This is the second one of a series of joint papers devoted to the construction of basic examples of codimension-one closed minimal submanifolds in spheres and compact symmetric spaces. In a previous paper [3], we constructed some new examples of codimension-one closed minimal submanifolds in $S^{3p+2}(1)$, and the following four symmetric spaces of rank 2, namely,

$$SU(3)/SO(3), \quad SU(3), \quad SU(6)/Sp(3), \quad E_6/F_4,$$

which are respectively of the diffeomorphic types of

$$S^1 \times S^p \times S^p \times S^p, \quad S^1 \times \frac{SO(3)}{\mathbf{Z}_2^2}, \quad S^1 \times \frac{SU(3)}{T^2},$$
$$S^1 \times \frac{Sp(3)}{Sp(1)^3}, \quad S^1 \times \frac{F_4}{\mathrm{Spin}(8)}.$$

In [3] we also ask the following problem of "generalized equator", namely,

Problem 1. Let M^n be a given compact symmetric space. Among all codimension-one submanifolds of M^n, which one is the "simplest" and the "best" that one may consider to be the generalized equator of M^n?

Of course, the above problem is as yet not precise because the "simplicity" as well as "value judgment" are purely a matter of taste and viewpoint. From differential topological viewpoint, spheres are the simplest and the most basic closed manifolds. Therefore it is rather natural to ask the following related problem of codimension-one minimal sphere.

Problem 2. Let M^n be a compact irreducible simply connected symmetric space. Are there codimension-one minimal imbeddings of the differentiable $(n-1)$-sphere into M^n?

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