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COMPLEX-ANALYTICITY OF HARMONIC MAPS, VANISHING AND LEFSCHETZ THEOREMS

YUM-TONG SIU

Dedicated to Professor S. S. Chern on his 70th birthday

Abstract

In this paper we discuss the relationship between the various techniques of proving vanishing theorems and the method of obtaining the complex-analyticity of harmonic maps between Kähler manifolds. We then obtain sharp results on the complex-analyticity of harmonic maps with the curvature conditions on the target manifold expressed in natural and familiar terms and also results concerning curvature characterizations of compact symmetric Kähler manifolds, Barth-Lefschetz type theorems, the generalization of the strong Lefschetz theorem, and vanishing theorems.

Introduction

This paper is an outgrowth of an attempt to apply the method of proving the strong rigidity of compact Kähler manifolds to obtain vanishing theorems for holomorphic vector bundles. To prove the strong rigidity of negatively curved compact Kähler manifolds, one tried to use harmonic maps $f: M \to N$ between compact Kähler manifolds (for definition and background of harmonic maps see [17], [18], or [53]) and the technique of considering $\Delta \mid \bar{\partial} f \mid^2$. (The technique of considering the Laplacian of the square norm was first introduced by Bochner [10] in the case of harmonic tensors on Riemannian manifolds and later applied by Kodaira [33] to $(0, q)$-forms on a Kähler manifold with values in a Hermitian holomorphic line bundle.) With this method of proving strong rigidity one encountered the difficulty of two curvature terms of opposite signs, one involving the Ricci curvature of M and the other involving the full curvature tensor of N. In [53] the author overcame this difficulty by the following variation of the Bochner-Kodaira technique which for convenience's sake we refer to in this paper as the $\partial\bar{\partial}$ Bochner-Kodaira technique. One considers the integral of $\partial\bar{\partial}(\Sigma_{\alpha,\beta}\, g_{\alpha\bar{\beta}}\bar{\partial} f^{\alpha} \wedge \partial f^{\bar{\beta}}) \wedge \omega_M^{n-2}$ over M instead of $\Delta \mid \bar{\partial} f \mid^2$, where $g_{\alpha,\bar{\beta}}$ is the Kähler metric of N, ω_M is the Kähler form of M, and

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