

FLAT LEFT-INVARIANT CONNECTIONS ADAPTED TO THE AUTOMORPHISM STRUCTURE OF A LIE GROUP

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Suppose that K is a Lie group with Lie algebra $\underline{K}$, and further that $\text{Aut}(\underline{K})$ (respectively $\text{Int}(\underline{K})$) is the group of automorphisms (resp. interior automorphisms) of the algebra $\underline{K}$. The local automorphism (resp. local interior automorphism) structure of K is the principal fiber bundle of frames obtained by the extension to $\text{Aut}(\underline{K})$ (resp. $\text{Int}(\underline{K})$) of a left-invariant parallelism of K . Its fibers are unique, up to a right translation in K 's frame bundle $R(K)$. In this article we commence a study of left-invariant locally flat connections adapted to the structures defined above.

INTRODUCTION. PRINCIPAL RESULTS

The problem of finding those Lie groups (necessarily solvable) which admit complete, locally flat (that is, of zero curvature and torsion), left invariant connections is an open problem (cf. J. Milnor [12]). In fact, few groups which possess such connections are known. One of the difficulties encountered while searching for necessary conditions for the existence of such connections is the fact that the relationship between the algebraic structure defined by the connection and that of the Lie algebra is not à priori, sufficiently strong to ensure any consequences for the group structure. Therefore it seems natural to consider, for a first approach to the problem, those connections which are more intrinsic—that is to say—those connections which are adapted to certain left-invariant G -structures over the group, where G is a linear group of automorphisms of the Lie algebra of the group under consideration. Suppose that K is a Lie group with Lie algebra $\underline{K}$, and further that $\text{Aut}(\underline{K})$ (respectively $\text{Int}(\underline{K})$) is the group of automorphisms (resp. interior automorphisms) of the algebra $\underline{K}$. The local automorphism (resp. local interior automorphism) structure of K is the principal fiber bundle of frames of K obtained by the extension

Communicated by A. Lichnerowicz, September 17, 1980. This article was written while the author was an Associate Professor at the Savoie University, France.