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FLOWS FOR DIFFERENTIABLE VECTOR FIELDS ON CONJUGATE BANACH SPACES

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Introduction

In [6] the author showed that many of the manifolds of maps which are used in global nonlinear analysis and previously had been regarded solely as infinite-dimensional Banach manifolds do in fact possess the much richer structure of objects which the author called Bw^* manifolds. The author introduced an approach to a theory of differential calculus for maps between Bw^* spaces as a tool for the development of differentiable Bw^* manifold theory, and isolated two classes of differentiable maps which the author labeled C^k and $\mathfrak{A}^k$. These classes of maps were shown to be closed under composition, and the inverse function theorem was established for $\mathfrak{A}^k$ maps.

The main result of this paper is Theorem 6.3, which will complete the development of differential calculus for $\mathfrak{A}^k$ maps by showing that $\mathfrak{A}^k$ vector fields generate $\mathfrak{A}^k$ flows. Since a Bw^* space can also be regarded as a Banach space, and since the $\mathfrak{A}^k$ vector fields represent a subclass of the usual C^k vector fields under this identification, this existence theorem for flows generated by $\mathfrak{A}^k$ vector fields is actually a strengthened version of the existence theorem for flows generated by C^k vector fields on Banach spaces. In fact, the ordinary existence theorem will be used in the proof of this new version in a manner reminiscent of the use of the ordinary inverse function in the proof of the inverse function theorem for $\mathfrak{A}^k$ maps.

Since a detailed account of the relevant basic theory of Bw^* spaces and differential calculus map be found in [6], the reader will be assumed to be acquainted with the material covered in that reference from the beginning of Chapter 2 through the first half of Chapter 6. However, for the sake of self-sufficiency, a brief review of essential information is included in §1.

1. Bw^* Spaces and Differential Calculus

Notation and terminology. Let X be a bw^* space. Then X' is a Banach space as is X'', and the natural linear injection of X into X'' is onto. Furthermore,

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