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EXAMPLES OF CODIMENSION-ONE CLOSED MINIMAL SUBMANIFOLDS IN SOME SYMMETRIC SPACES. I

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1. Introduction

In the study of Riemannian geometry, the symmetric spaces constitute a natural family of nice testing spaces. They are characterized by a single neat property of being "symmetric with respect to any point" and, rather remarkably, can be classified via the structure-classification theory of semi-simple Lie groups [É. Cartan]. Therefore the study of geometry of submanifolds in symmetric spaces is a natural generalization to that of spaces of constant curvature which provides an ideal setting for in-depth investigation of the interaction between geometry and Lie group theory. However, such problems have been so far almost left unexplored. Hence let us begin with formulating some simple problems along the above lines.

In the case of compact symmetric spaces, the spheres S^n is one of the simplest and also the most well understood of Riemannian manifolds. Among all submanifolds of a given dimension $1 \leqslant r \leqslant n-1$ in S^n, the equator S^r is clearly the simplest and the "best" one. Therefore it is natural to pose the following problem

Problem 1. Let M^n be a given compact symmetric space. Among all r-dimensional submanifolds of M^n, $1 \leqslant r \leqslant n-1$, which one is the "simplest" and the "best" that one may consider it to be the "generalized r-dimensional equator" in M^n?

Of course, the above problem is as yet not precise because the "simplicity" and the "virtue" of submanifolds is, in fact, purely a matter of taste. Therefore one may adopt different "standards" to get possibly different generalized equators. For example, it is not too difficult to prove that the r-dimensional equator S^r is the unique closed r-dimensional minimal submanifold with the least total (r-dimensional) volume. Therefore the following precise problem is a natural variant of problem 1.

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