INTEGRAL FORMULAS FOR SUBMANIFOLDS AND THEIR APPLICATIONS

D. J. SHETTY & KRISHNA AMUR

Introduction

Integral formulas of Minkowski type have been studied and applied in characterizing umbilical submanifolds by Chen [3], Katsurada [5], [6], [7], Kôjyô [6], Nagai [7], Okumara [10], Tani [11] and Yano [3], [8], [9], [10], [11]. These authors assumed that the normal vector field e with respect to which the integral formulas were obtained was parallel in the normal bundle[1]. The purpose of this paper is to extend the study of the above authors. We obtain the most general integral formulas for a submanifold of a Riemannian space of constant sectional curvature without putting any restriction on the unit normal vector field e, and under conditions which are weaker than the condition that e be parallel in the normal bundle we obtain integral formulas of Minkowski type and apply them to the study of umbilical submanifolds. We give concrete illustrations to substantiate our generalisations.

1. Preliminaries

Let M be an orientable differentiable manifold of dimension n imbedded in an orientable m-dimensional Riemannian manifold N of constant sectional curvature. Let $u^a = u^a(x^h)$ denote the local expression of the submanifold M in N. Here and in the sequel $a, b, c, \cdots$ run over the range $1, 2, \cdots, m$, and $h, i, j, \cdots$ over the range $1, 2, \cdots, m$ unless otherwise specified. We shall identify vector fields of M with their images under the differential mapping. Thus if X is a vector field of M and has local expression $X = X^h \partial_h$, then it has local expression $X = X^h B_h^a \partial_a$ in N where $\partial_h = \partial/\partial x^h$, $\partial_a = \partial/\partial u^a$, $B_h^a = \partial u^a/\partial x^h$, and Einstein's summation convention is followed for repeated

[1] For a generalization of the results of these authors see C. C. Hsiung, J. D. Liu and S. S. Mittra, *Integral formulas for closed submanifolds of a Riemannian manifold*, J. Differential Geometry **12** (1977) 133–151, which was published after the present paper had been written.