

SOME NEW RIEMANNIAN INVARIANTS

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Introduction

The purpose of this paper is to introduce some new riemannian invariants and to study their properties. In a future paper we will study riemannian manifolds whose invariants are large.

In the first section the invariants are defined and are related to the dimension of the group of isometries. In particular, we have

$$\dim I_p \leq \frac{1}{2}ATC_p(2n - ATC_p - 1),$$

where I_p is the isotropy group of isometries at a point p of an n -dimensional complete connected riemannian manifold M , and ATC_p is one of the invariants.

In the second section we show, using the invariants and the Rauch comparison theorem, that for manifolds whose diameter is small relative to their sectional curvature, the group I_p is finite for all p in M . We also study other properties of such “small diameter” manifolds.

In the third section we study how the invariants behave under products and coverings.

In the fourth section we compute the invariants on some riemannian manifolds.

In the fifth section we study in detail some of the properties the invariants possess. In particular we study the p -dependence.

In the sixth section we prove a result which relates the geometries of the submanifolds in question.

Throughout the paper a manifold will be a complete connected riemannian manifold unless otherwise stated. A submanifold will always be an embedded submanifold.

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