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POSITIVE RICCI CURVATURE ON FIBRE BUNDLES

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In this paper we construct complete metrics of positive Ricci curvature on a large class of fibre bundles. Some of the results for compact fibres have been obtained independently by Poor [12]. The base manifold M is assumed to be compact admitting a metric with $\mathrm{Ric}_M > 0$. If $F = G/H$ is compact homogeneous with $\pi_1(F)$ finite, we show that any bundle over M with fibre F admits a metric with $\mathrm{Ric} > 0$. Certain exotic 7- and 15-spheres arise as sphere bundles over spheres and, thus, admit metrics of positive Ricci curvature.

For vector bundles we have the following result.

Theorem. *Let $\hat{\pi}$: $B \to M$ be a vector bundle over M, a compact manifold admitting a metric of positive Ricci curvature. If the fibre dimension is greater than two, B admits a complete metric of positive Ricci curvature.*

This result is related to a question of Cheeger and Gromoll [1]: Does any vector bundle over S^n admit a complete metric with $K \geqslant 0$? Rigas has some partial results on this problem [13].

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1. Preliminaries

We begin by recalling some basic notions and introducing notation. All objects (manifolds, maps, actions, etc.) will be C^∞, and M^n denotes a manifold of dimension n. The differential of a map f: $M \to N$ between manifolds will usually be abbreviated to $f_p(X)$ of just $f(X)$ for $X \in T_pM$. For a Riemannian manifold M we use the following curvature convention:

$$R_M(X, Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z,$$

$$R_M(X, Y, Z, W) = \langle R_M(X, Y)Z, W\rangle,$$

$$R_M(X, Y) = R_M(X, Y, Y, X).$$

$\mathfrak{X}(M)$ denotes the C^∞ vector fields on M.

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