J. DIFFERENTIAL GEOMETRY
14 (1979) 81-92

KAEHLERIAN MANIFOLDS WITH CONSTANT SCALAR CURVATURE ADMITTING A HOLOMORPHICALLY PROJECTIVE VECTOR FIELD

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To Professor C. C. Hsiung on his sixtieth birthday

1. Introduction

Let M be a connected Kaehlerian manifold of complex dimension n covered by a system of real coordinate neighborhoods $\{U; x^h\}$, where, here and in the sequel the indices $h, i, j, k, \ldots$ run over the range $\{1, 2, \ldots, 2n\}$, and let g_{ji}, $F_i^{\ h}$, $\{_j{}^h{}_i\}$, ∇_i, $K_{kji}{}^h$, K_{ji} and K be the Hermitian metric tensor, the complex structure tensor, the Christoffel symbols formed with g_{ji}, the operator of covariant differentiation with respect to $\{_j{}^h{}_i\}$, the curvature tensor, the Ricci tensor and the scalar curvature of M respectively.

A vector field v^h is called a *holomorphically projective* (or *H-projective*, for brevity) vector field [1], [2], [5] if it satisfies

$$(1.1)\qquad \mathfrak{L}_v\{_j{}^h{}_i\} = \nabla_j\nabla_i v^h + v^k K_{kji}{}^h = \rho_j\delta_i^h + \rho_i\delta_j^h - \rho_s F_j^{\ s}F_i^{\ h} - \rho_s F_i^{\ s}F_j^{\ h}$$

for a certain covariant vector field ρ_j on M called the *associated* covariant vector field of v^h, where $\mathfrak{L}_v$ denotes the operator of Lie derivation with respect to v^h. In particular, if ρ_j is the zero-vector field, then v^h is called an *affine* vector field.

When we refer in the sequel to an H-projective vector field v^h, we always mean by ρ_j the associated covariant vector field appearing in (1.1).

In the present paper, we first prove a series of integral inequalities in a Kaehlerian manifold with constant scalar curvature admitting an H-projective vector field, and then find necessary and sufficient conditions for such a Kaehlerian manifold to be isometric to a complex projective space with Fubini-Study metric.

In the sequel, we need the following theorem due to Obata [4]. (See also [3].)

Theorem A. *Let M be a complete connected and simply connected Kaehlerian manifold. In order for M to admit a nontrivial solution φ of a system*

Communicated January 13, 1977.