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COMPACT REAL HYPERSURFACES OF A COMPLEX PROJECTIVE SPACE

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Introduction

Let M be an n-dimensional real hypersurface of a complex projective space $CP^{(n+1)/2}$ of complex dimension $(n+1)/2$, and H the Weingarten map of the immersion $i: M \to CP^{(n+1)/2}$. It is known [1] that if a compact minimal hypersurface M of $CP^{(n+1)/2}$ satisfies trace $H^2 \leq n-1$, then trace $H^2 = n-1$, and up to isometries of $CP^{(n+1)/2}$, M is a certain distinguished minimal hypersurface $M^c_{p,q}$ for some p and q.

The purpose of the present paper is to generalize the above result in such a way that we have an integral inequality which is still valid even if the immersion i is not necessarily minimal. Two main tools for this purpose are Lemma 1.1, to be stated in § 1, and the following integral formula established by Yano [3], [4]:

$$(0.1) \qquad \int_M \{\mathrm{Ric}\,(X,X) + \tfrac{1}{2}|L(X)g|^2 - |\nabla X|^2 - (\mathrm{div}\, X)^2\}{*1} = 0\ ,$$

where X is an arbitrary tangent vector field on M, $*1$ is the volume element of M, and $|Y|$ denotes the length with respect to the Riemannian metric of a vector field Y on M.

In § 1 we explain the model space $M^c_{p,q}$, and in § 2 we present some formulas to be used in § 3. Finally in § 3 we prove our main result.

1. Submersion, immersion and the model $M^c_{p,q}$

Let S^{n+2} be an odd-dimensional sphere of radius 1 in a Euclidean $(n+3)$-space E^{n+3}, $CP^{(n+1)/2}$ the complex projective space, and $\tilde{\pi}$ the Riemannian submersion with totally geodesic fibres, which is defined by the Hopf fibration $S^{n+2} \to CP^{(n+1)/2}$. The almost complex structure J of $CP^{(n+1)/2}$ is nothing but the fundamental tensor of the submersion $\tilde{\pi}$, and the Riemannian metric G of $CP^{(n+1)/2}$ is induced naturally from that of S^{n+2}. With respect to (J, G), $CP^{(n+1)/2}$ is a Kaehlerian manifold of constant holomorphic sectional curvature 4. The curvature tensor $\bar{R}$ of $CP^{(n+1)/2}$ is given by

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