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THE LENGTH SPECTRA OF SOME COMPACT MANIFOLDS OF NEGATIVE CURVATURE

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1. Introduction

Let R be a compact Riemannian manifold. In each free homotopy class $\tilde{\gamma}$ of closed paths on R, there exists a geodesic whose length is minimal among the paths in $\tilde{\gamma}$; let $l(\tilde{\gamma})$ be its length. The distinct members of the set of lengths $l(\tilde{\gamma})$ as $\tilde{\gamma}$ varies over all such classes can be arranged in increasing order $0 < l_1 < l_2 < \cdots$. The sequence $\{l_i\}_{i\geq 1}$, finite or infinite, is by definition the length spectrum of R. It may happen that $l(\tilde{\gamma}) = l(\tilde{\gamma}')$ for two distinct classes. Let, for each $i \geq 1$, m_i be the number of free homotopy classes $\tilde{\gamma}$ such that $l(\tilde{\gamma}) = l_i$. The sequence $\{(l_i, m_i)\}_{i\geq 1}$ may be called the length spectrum with multiplicity.

Let Δ be the Laplace-Beltrami operator of R. Then the space $L_2(R)$ (with respect to the Riemannian measure) decomposes as the Hilbert space direct sum of finite dimensional eigenspaces for Δ. Let $\{\lambda_i\}_{i\geq 1}$ be the distinct eigenvalues, and n_i the multiplicity of λ_i. The sequence $\{(\lambda_i, n_i)\}_{i\geq 1}$ is the spectrum of Δ. We may assume the λ_i to be arranged so that $0 \geq \lambda_1 > \lambda_2 > \cdots$.

In this paper, we shall study the length spectrum and its relation to the spectrum of Δ for a very special type of compact manifold of negative sectional curvature. Specifically, we shall consider a compact manifold R whose simply connected Riemannian covering manifold H is a symmetric space of noncompact type and of rank 1. As is well-known, H can then be represented as G/K, where G is a noncompact connected simple Lie group of R-rank one, with finite center, and K is a maximal compact subgroup of G. As a consequence R can be represented as $\Gamma\backslash G/K$, where Γ is a discrete subgroup of G, acting freely on G/K, such that $\Gamma\backslash G$ is compact. Γ can be identified with the fundamental group of R. The metric on R is fixed to be the one obtained from the canonical G-invariant metric on G/K. Cf. [11], [27].

For such a manifold R, let $\{(l_i, m_i)\}_{i\geq 1}$ be the length spectrum with multiplicity, and for any $l \geq 0$, define $Q_1(l) = \sum_{\{i; l_i \leq l\}} m_i$. Thus $Q_1(l)$ is the number of free homotopy classes $\tilde{\gamma}$ such that $l(\tilde{\gamma}) \leq l$. It can be seen easily that $Q_1(l)$ is finite for each finite l. We shall show that the asymptotic behaviour of $Q_1(l)$ as $l \to \infty$ can be described precisely in terms of the covering space G/K. In fact, we find that $Q_1(l) \sim (2|\rho| l)^{-1} \exp 2|\rho| l$ as $l \to \infty$, where ρ is the half

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