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A CLASS OF COMPLEX ANALYTIC FOLIATE MANIFOLDS WITH RIGID STRUCTURE

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In 1957, R. Bott [1] proved that the complex projective spaces have a rigid complex structure. On the other hand, in 1961 Kodaira and Spencer [9] extended the deformation theory to general multifoliate complex structures and, particularly, to complex analytic foliations. But, so far as we know, no example of a rigid structure of this kind has been provided. It is our aim here to prove the rigidity of a class of complex foliate manifolds which generalizes the complex projective spaces. Our class contains as a particular case any product of two complexprojective spaces.

The complex manifolds under consideration will be compact Kählerian, the result being obtained by the general method initiated by Bochner, which consists in studying the relations between curvature and cohomology. Namely, we shall go along the lines of Calabi-Vesentini's paper [3] to prove first a generalized Nakano inequality. In connection with our previous cohomology calculations of [13], [14], this will lead to the desired results.

Some other related remarks will also be made.

1. A complex analytic foliate (c.a.f.) structure $\mathscr{F}$ of complex codimension n on a complex $(n+m)$-dimensional manifold V is given by an atlas $\{U_\alpha; z_\alpha^a, z_\alpha^u\}$ $(a, b, \cdots = 1, \cdots, n; u, v, \cdots = n+1, \cdots, n+m)$, such that on $U_\alpha \cap U_\beta \neq \emptyset$ one has, besides analyticity,

$$\partial z_\beta^a / \partial z_\alpha^u = 0 \,. \tag{1.1}$$

Then the maximal connected submanifolds which can be represented locally by $z_\alpha^a =$ const. are the *leaves* of $\mathscr{F}$, and the images $\varphi_\alpha(U_\alpha) \subset C^n$ of the submersions $\varphi_\alpha: U_\alpha \to C^n$ defined by $\varphi_\alpha(z_\alpha^a, z_\alpha^u) = (z_\alpha^a)$ (C is the complex line) are called the *local transverse manifolds*.

The tangent vectors of the leaves define the *structural subbundle* F of $T(V)$ with local bases $Z_u = \partial/\partial z_\alpha^u$ and transition functions $(\partial z_\beta^u/\partial z_\alpha^v)$. $T(V)/F = F'$ is the *transversal bundle* with the local bases defined by the equivalence classes $[\partial/\partial z_\alpha^a]$ and the transition functions $(\partial z_\beta^a/\partial z_\alpha^b)$.

Generally, we shall say that the elements depending only on the leaves are *foliate* and, particularly, c.a.f. For instance, $f: V \to C$ is foliate if $\partial f/\partial z_\alpha^u =$

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