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ON TWO NOTIONS OF STRUCTURAL STABILITY

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Introduction

In the literature one can find two notions of structural stability. First the original one given by Andronov and Pontriagin (see [1], [2], [5]) stated for vector fields, that is, for the actions of the additive group of real number $\boldsymbol{R}$ on a manifold M. This definition says roughly that an $\boldsymbol{R}$-action on M is structurally stable if, for any other $\boldsymbol{R}$-action close-to, in the sense that the vector fields generating these actions are close, there exists a homeomorphism of M onto itself mapping the orbits of the first action onto the orbits of the second. This definition can readily be extended (see below § 1), to actions on M of a given real Lie group G in particular $G = Z =$ additive group of all integers.

Another definition was proposed more recently by Smale (see [8] and [9]) for Z-actions on M. Such an action is generated by an diffeomorphism $\phi: M \to M$. Smale's definition is roughly that ϕ is structurally stable if any diffeomorphism ψ sufficiently close to ϕ in the C^1-topology is topologically conjugate to ϕ. Smale's definition, which can also be extended to action on M of any given real Lie group G, seems more restrictive than the one of Andronov and Pontrjagin.

The purpose of this note is to show that in the case $G = Z$, the two definitions are equivalent if the dimension of $M > 1$ and M is connected.

In § 1 we give precise statements of the two definitions, first in the case $G = Z$ (which is the one of interest to us) and then in the general case, for comparison sake.

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1. Definitions of structural stability

A C^1-Z-action on a compact C^∞ manifold M is generated by a C^1-diffeomorphism $\phi: M \to M$.

Definition 1 (*Andronov-Pontrjagin*). A C^1-diffeomorphism $\phi: M \to M$ is structurally stable if for any $\varepsilon > 0$ there exists a neighborhood U of ϕ in

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